

Valuation and Capital Budgeting for the Levered Firm

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EXECUTIVE SUMMARY

Instructors often structure the basic course in corporate finance around the two sides of the balance sheet. The left-hand side of the balance sheet contains assets. Chapters 4, 5, 6, 7, and 8 of this textbook treat the capital-budgeting decision, which is a decision concerning the assets of the firm. Chapters 9, 10, 11, and 12 treat the discount rate for a project, so those chapters also concern the left-hand side of the balance sheet. The right-hand side of the balance sheet contains liabilities and owner's equity. Chapters 13, 14, 15, and 16 of this textbook examine the debt-versus-equity decision, which is a decision about the right-hand side of the balance sheet.

While the preceding chapters of this textbook have, for the most part, treated the capital-budgeting decision separately from the capital-structure decision, the two decisions are actually related. As we will see, a project of an all-equity firm might be rejected, while the same project might be accepted for a levered but otherwise identical firm. This occurs because the cost of capital frequently decreases with leverage, thereby turning some negative NPV projects into positive NPV projects.

Chapters 4 through 8 implicitly assumed that the firm is financed with only equity. The goal of this chapter is to value a project, or the firm itself, when leverage is employed. We point out that there are three standard approaches to valuation under leverage: the adjusted-present-value (APV) method, the flow-to-equity (FTE) method, and the weighted-average-cost-of-capital (WACC) method. These three approaches may seem, at first glance, to be quite different. However, we show that, if applied correctly, all three approaches provide the same value estimate.

The three methods discussed next can be used to value either the firm as a whole or a project. The example below discusses project value, though everything we say applies to an entire firm as well.

17.1 ADJUSTED-PRESENT-VALUE APPROACH

The **adjusted-present-value (APV)** method is best described by the following formula:

$$\text{APV} = \text{NPV} + \text{NPVF}$$

In words, the value of a project to a levered firm (APV) is equal to the value of the project to an unlevered firm (NPV) plus the net present value of the financing side effects (NPVF). One can generally think of four side effects:

1. *The Tax Subsidy to Debt.* This was discussed in Chapter 15, where we pointed out that, for perpetual debt, the value of the tax subsidy is $T_c B$. (T_c is the corporate tax rate, and B is the value of the debt.) The material on valuation under corporate taxes in Chapter 15 is actually an application of the APV approach.

2. *The Costs of Issuing New Securities.* As we will discuss in detail in Chapter 20, investment bankers participate in the public issuance of corporate debt. These bankers must be compensated for their time and effort, a cost that lowers the value of the project.
3. *The Costs of Financial Distress.* The possibility of financial distress, and bankruptcy in particular, arises with debt financing. As stated in the previous chapter, financial distress imposes costs, thereby lowering value.
4. *Subsidies to Debt Financing.* The interest on debt issued by state and local governments is not taxable to the investor. Because of this, the yield on tax-exempt debt is generally substantially below the yield on taxable debt. Frequently, corporations are able to obtain financing from a municipality at the tax-exempt rate because the municipality can borrow at that rate as well. As with any subsidy, this subsidy adds value.

While each of the preceding four side effects is important, the tax deduction to debt almost certainly has the highest dollar value in practice. For this reason, the following example considers the tax subsidy, but not the other three side effects.¹

Consider a project of the P. B. Singer Co. with the following characteristics:

Cash inflows: \$500,000 per year for the indefinite future

Cash costs: 72% of sales

Initial investment: \$475,000

$T_C = 34\%$

$r_0 = 20\%$, where r_0 is the cost of capital for a project of an all-equity firm.

If both the project and the firm are financed with only equity, the project's cash flow is

Cash inflows	\$500,000
Cash costs	<u>−360,000</u>
Operating income	140,000
Corporate tax (.34 tax rate)	<u>−47,600</u>
Unlevered cash flow (UCF)	\$92,400

The distinction in Chapter 4 between present value and net present value is important for this example. As pointed out in Chapter 4, the *present value* of a project is determined before the initial investment at date 0 is subtracted. The initial investment is subtracted for the calculation of *net* present value.

Given a discount rate of 20 percent, the present value of the project is

$$\frac{\$92,400}{0.20} = \$462,000$$

The net present value (NPV) of the project, that is, the value of the project to an all-equity firm, is

$$\$462,000 - \$475,000 = -\$13,000$$

Since the NPV is negative, the project would be rejected by an all-equity firm.

Now imagine that the firm finances the project with exactly \$126,229.50 in debt, so that the remaining investment of \$348,770.50 ($\$475,000 - \$126,229.50$) is financed with equity. The net present value of the project under leverage, which we call the adjusted present value, or the APV, is

$$\begin{aligned} \text{APV} &= \text{NPV} + T_C \times B \\ \$29,918 &= -\$13,000 + 0.34 \times \$126,229.50 \end{aligned}$$

¹The Bicksler Enterprises example of Section 17.6 handles both flotation costs and interest subsidies.

That is, the value of the project when financed with some leverage is equal to the value of the project when financed with all equity plus the tax shield from the debt. Since this number is positive, the project should be accepted.

You may be wondering why we chose such a precise amount of debt. Actually, we chose it so that the ratio of debt to the present value of the project under leverage is 0.25.²

In this example, debt is a fixed proportion of the present value of the project, not a fixed proportion of the initial investment of \$475,000. This is consistent with the goal of a target debt-to-market-value ratio, which we find in the real world. For example, commercial banks typically lend to real estate developers a fixed percentage of the market value of a project, not a fixed percentage of the initial investment.

CONCEPT
QUESTIONS
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- How is the APV method applied?
- What additional information beyond NPV does one need to calculate APV?

17.2 FLOW-TO-EQUITY APPROACH

The **flow-to-equity (FTE)** approach is an alternative capital-budgeting approach. The formula simply calls for discounting the cash flow from the project to the equityholders of the levered firm at the cost of equity capital, r_S . For a perpetuity, this becomes

$$\frac{\text{Cash flow from project to equityholders of the levered firm}}{r_S}$$

There are three steps to the FTE approach.

Step 1: Calculating Levered Cash Flow (LCF)³

Assuming an interest rate of 10 percent, the perpetual cash flow to equityholders in our example is

Cash inflows	\$500,000.00
Cash costs	−360,000.00
Interest (10% × \$126,229.50)	−12,622.95
Income after interest	127,377.05
Corporate tax (.34 tax rate)	−43,308.20
Levered cash flow (LCF)	\$ 84,068.85

²That is, the present value of the project after the initial investment has been made is \$504,918 (\$29,918 + \$475,000). Thus, the debt-to-value ratio of the project is 0.25 (\$126,229.50/\$504,918).

This level of debt can be calculated directly. Note that

$$\begin{aligned} \text{Present value of levered project} &= \text{Present value of unlevered project} + T_C \times B \\ V_{\text{With debt}} &= \$462,000 + 0.34 \times .25 \times V_{\text{With debt}} \end{aligned}$$

Rearranging the last line, we have

$$\begin{aligned} V_{\text{With debt}}(1 - 0.34 \times 0.25) &= \$462,000 \\ V_{\text{With debt}} &= \$504,918 \end{aligned}$$

Since debt is 0.25 of value, debt is \$126,229.50 (0.25 × \$504,918).

³We use the term *levered cash flow (LCF)* for simplicity. A more complete term would be *cash flow from the project to the equityholders of a levered firm*. Similarly, a more complete term for *unlevered cash flow (UCF)* would be *cash flow from the project to the equityholders of an unlevered firm*.

Alternatively, one can calculate levered cash flow (LCF) directly from unlevered cash flow (UCF). The key here is that the difference between the cash flow that equityholders receive in an unlevered firm and the cash flow that equityholders receive in a levered firm is the after-tax interest payment. (Repayment of principal does not appear in this example, since the debt is perpetual.) One writes this algebraically as

$$\text{UCF} - \text{LCF} = (1 - T_C)r_B B$$

The term on the right-hand side of this expression is the after-tax interest payment. Thus, since cash flow to the unlevered equityholders (UCF) is \$92,400 and the after-tax interest payment is \$8,331.15 $[(.66) \cdot 10 \times \$126,229.50]$, cash flow to the levered equityholders (LCF) is

$$\$92,400 - \$8,331.15 = \$84,068.85$$

which is exactly the number we calculated earlier.

Step 2: Calculating r_S

The next step is to calculate the discount rate, r_S . Note that we assumed that the discount rate on unlevered equity, r_0 , is .20. As we saw in Chapter 15, the formula for r_S is

$$r_S = r_0 + \frac{B}{S}(1 - T_C)(r_0 - r_B)$$

Note that our target debt-to-value ratio of 1/4 implies a target debt-to-equity ratio of 1/3. Applying the preceding formula to this example, we have

$$r_S = .222 = .20 + \frac{1}{3}(.66)(.20 - .10)$$

Step 3: Valuation

The present value of the project's LCF is

$$\frac{\text{LCF}}{r_S} = \frac{\$84,068.85}{.222} = \$378,688.50$$

Since the initial investment is \$475,000 and \$126,299.50 is borrowed, the firm must advance the project \$348,770.50 $(\$475,000 - \$126,299.50)$ out of its own cash reserves. The *net* present value of the project is simply the difference between the present value of the project's LCF and the investment not borrowed. Thus, the NPV is

$$\$378,688.50 - \$348,770.50 = \$29,918$$

which is identical to the result found with the APV approach.



- How is the FTE method applied?
- What information is needed to calculate FTE?

17.3 WEIGHTED-AVERAGE-COST-OF-CAPITAL METHOD

Finally, one can value a project using the **weighted-average-cost-of-capital (WACC)** method. While this method was discussed in Chapters 12 and 15, it is worthwhile to review it here. The WACC approach begins with the insight that projects of levered firms are

simultaneously financed with both debt and equity. The cost of capital is a weighted average of the cost of debt and the cost of equity. As seen in Chapters 12 and 15, the cost of equity is r_S . Ignoring taxes, the cost of debt is simply the borrowing rate, r_B . However, with corporate taxes, the appropriate cost of debt is $(1 - T_C)r_B$, the after-tax cost of debt.

The formula for determining the weighted average cost of capital, r_{WACC} , is

$$r_{WACC} = \frac{S}{S+B} r_S + \frac{B}{S+B} r_B (1 - T_C)$$

The weight for equity, $\frac{S}{S+B}$, and the weight for debt, $\frac{B}{S+B}$, are target ratios. Target ratios are generally expressed in terms of market values, not accounting values. (Recall that another phrase for accounting value is *book value*.)

The formula calls for discounting the *unlevered* cash flow of the project (UCF) at the weighted average cost of capital, r_{WACC} . The net present value of the project can be written algebraically as

$$\sum_{t=1}^{\infty} \frac{UCF_t}{(1 + r_{WACC})^t} - \text{Initial investment}$$

If the project is a perpetuity, the net present value is

$$\frac{UCF}{r_{WACC}} - \text{Initial investment}$$

We previously stated that the target debt-to-value ratio of our project is 1/4 and the corporate tax rate is .34, implying that the weighted average cost of capital is

$$r_{WACC} = \frac{3}{4} \times 0.222 + \frac{1}{4} \times 0.10(0.66) = 0.183$$

Note that r_{WACC} , 0.183, is lower than the cost of equity capital for an all-equity firm, 0.20. This must always be the case, since debt financing provides a tax subsidy that lowers the average cost of capital.

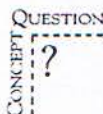
We previously determined the UCF of the project to be \$92,400, implying that the present value of the project is

$$\frac{\$92,400}{0.183} = \$504,918$$

Since this initial investment is \$475,000, the NPV of the project is

$$\$504,918 - \$475,000 = \$29,918$$

In this example, all three approaches yield the same value.



- How is the WACC method applied?

17.4 A COMPARISON OF THE APV, FTE, AND WACC APPROACHES

Capital-budgeting techniques in the early chapters of this text applied to all-equity firms. Capital budgeting for the levered firm could not be handled early in the book because the effects of debt on firm value were deferred until the previous two chapters. We learned there that debt increases firm value through tax benefits but decreases value through bankruptcy and related costs.

In this chapter we provide three approaches to capital budgeting for the levered firm. The adjusted-present-value (APV) approach first values the project on an all-equity basis. That is, the project's after-tax cash flows under all-equity financing (called unlevered cash flows, or UCF) are placed in the numerator of the capital-budgeting equation. The discount rate, assuming all-equity financing, appears in the denominator. At this point, the calculation is identical to that performed in the early chapters of this book. We then add the net present value of the debt. We point out that the net present value of the debt is likely to be the sum of four parameters: tax effects, flotation costs, bankruptcy costs, and interest subsidies.

The flow-to-equity (FTE) approach discounts the after-tax cash flow from a project going to the equityholders of a levered firm (LCF). LCF, which stands for levered cash flow, is the residual to equityholders after interest has been deducted. The discount rate is r_S , the cost of capital to the equityholders of a levered firm. For a firm with leverage, r_S must be greater than r_0 , the cost of capital for an unlevered firm. This follows from our material in Chapter 15 showing that leverage raises the risk to the equityholders.

The last approach is the weighted-average-cost-of-capital (WACC) method. This technique calculates the project's after-tax cash flows assuming all-equity financing (UCF). The UCF is placed in the numerator of the capital-budgeting equation. The denominator, r_{WACC} , is a weighted average of the cost of equity capital and the cost of debt capital. The tax advantage of debt is reflected in the denominator because the cost of debt capital is determined net of corporate tax. The numerator does not reflect debt at all.

All three approaches perform the same task: valuation in the presence of debt financing. And, as illustrated by the previous example, all three provide the same valuation estimate. However, as we saw before, the approaches are markedly different in technique. Because of this, students often ask questions of the following sort: "How can this be? How can the three approaches look so different and yet give the same answer?" We believe that the best way to handle questions like these is through the following two points.

1. *APV versus WACC.* Of the three approaches, APV and WACC display the greatest similarity. After all, both approaches put the unlevered cash flow (UCF) in the numerator. However, the APV approach discounts these flows at r_0 , yielding the value of the unlevered project. Adding the present value of the tax shield gives the value of the project under leverage. The WACC approach discounts UCF at r_{WACC} , which is lower than r_0 .

Thus, both approaches adjust the basic NPV formula for unlevered firms in order to reflect the tax benefit of leverage. The APV approach makes this adjustment directly. It simply adds in the present value of the tax shield as a separate term. The WACC approach makes the adjustment in a more subtle way. Here, the discount rate is lowered below r_0 . Although we do not provide a proof in the textbook, it can be shown that these two adjustments always have the same quantitative effect.

2. *Entity Being Valued.* The FTE approach appears at first glance to be far different from the other two. For both the APV and the WACC approaches, the initial investment is subtracted out in the final step (\$475,000 in our example). However, for the FTE approach, only the firm's contribution to the initial investment (\$348,770.50 = \$475,000 - \$126,229.50) is subtracted out. This occurs because under the FTE approach, only the future cash flows to the levered equityholders (LCF) are valued. By contrast, future cash flows to the unlevered equityholders (UCF) are valued in both the APV and WACC approaches. Thus, since LCFs are net of interest payments, whereas UCFs are not, the initial investment under the FTE approach is correspondingly reduced by debt financing. In this way, the FTE approach produces the same answer that the other two approaches do.

A Suggested Guideline

The net present value of our project is exactly the same under each of the three methods. In theory, this should always be the case.⁴ However, one method usually provides an easier computation than another, and, in many cases, one or more of the methods are virtually impossible computationally. We first consider when it is best to use the WACC and FTE approaches.

If the risk of a project stays constant throughout its life, it is plausible to assume that r_0 remains constant throughout the project's life. This assumption of constant risk appears to be reasonable for most real-world projects. In addition, if the debt-to-value ratio remains constant over the life of the project, both r_S and r_{WACC} will remain constant as well. Under this latter assumption, either the FTE or the WACC approach is easy to apply. However, if the debt-to-value ratio varies from year to year, both r_S and r_{WACC} vary from year to year as well. Using the FTE or the WACC approach when the denominator changes every year is computationally quite complex, and when computations become complex, the error rate rises. Thus, both the FTE and WACC approaches present difficulties when the debt-to-value ratio changes over time.

The APV approach is based on the *level* of debt in each future period. Consequently, when the debt level can be specified precisely for future periods, the APV approach is quite easy to use. However, when the debt level is uncertain, the APV approach becomes more problematic. For example, when the debt-to-value ratio is constant, the debt level varies with the value of the project. Since the value of the project in a future year cannot be easily forecast, the level of debt cannot be easily forecast either.

Thus, we suggest the following guideline:

Use WACC or FTE if the firm's target debt-to-value *ratio* applies to the project over its life.
Use APV if the project's *level* of debt is known over the life of the project.

There are a number of situations where the APV approach is preferred. For example, in a leveraged buyout (LBO) the firm begins with a large amount of debt but rapidly pays down the debt over a number of years. Since the schedule of debt reduction in the future is known when the LBO is arranged, tax shields in every future year can be easily forecast. Thus, the APV approach is easy to use here. (An illustration of the APV approach applied to LBOs is provided in the appendix to this chapter.) By contrast, the WACC and FTE approaches are virtually impossible to apply here, since the debt-to-equity value cannot be expected to be constant over time. In addition, situations involving interest subsidies and flotation costs are much easier to handle with the APV approach. (The Bicksler Enterprises

⁴See I. Inselbag and H. Kaufold, "Two DCF Approaches for Valuing Companies under Alternative Financial Strategies (and How to Choose Between Them)" *Journal of Applied Corporate Finance* (Spring 1997).

THE THREE METHODS OF CAPITAL BUDGETING WITH LEVERAGE

1. Adjusted-Present-Value (APV) Method

$$\sum_{t=1}^{\infty} \frac{UCF_t}{(1 + r_0)^t} + \text{Additional effects of debt} - \text{Initial investment}$$

UCF_t = The project's cash flow at date t to the equityholders of an unlevered firm
 r_0 = Cost of capital for project in an unlevered firm

2. Flow-to-Equity (FTE) Method

$$\sum_{t=1}^{\infty} \frac{LCF_t}{(1 + r_S)^t} - (\text{Initial investment} - \text{Amount borrowed})$$

LCF_t = The project's cash flow at date t to the equityholders of a levered firm
 r_S = Cost of equity capital with leverage

3. Weighted-Average-Cost-of-Capital (WACC) Method

$$\sum_{t=1}^{\infty} \frac{UCF_t}{(1 + r_{WACC})^t} - \text{Initial investment}$$

r_{WACC} = Weighted average cost of capital

Notes:

1. The middle term in the APV formula implies that the value of a project with leverage is greater than the value of the project without leverage. Since $r_{WACC} < r_0$, the WACC formula implies that the value of a project with leverage is greater than the value of the project without leverage.
2. In the FTE method, cash flow *after interest* (LCF) is used. Initial investment is reduced by *amount borrowed* as well.

Guidelines:

1. Use WACC or FTE if the firm's target debt-to-value *ratio* applies to the project over its life.
2. Use APV if the project's *level of debt* is known over the life of the project.

example in Section 17.6 applies the APV approach to subsidies and flotation costs.) Finally, the APV approach handles the lease-versus-buy decision much more easily than does either the FTE or the WACC approach. (A full treatment of the lease-versus-buy decision appears in Chapter 24.)

The preceding examples are special cases. Typical capital budgeting situations are more amenable to either the WACC or the FTE approach than to the APV approach. Financial managers generally think in terms of target debt-value *ratios*. If a project does better than expected, both its value and its debt capacity will likely rise. The manager will increase debt correspondingly here. Conversely, the manager would be likely to reduce debt if the value of the project were to decline unexpectedly. Of course, because financing is a time-consuming task, the ratio cannot be adjusted on a day-to-day or a month-to-month basis. Rather, the adjustment can be expected to occur over the long run. As mentioned before, the WACC and FTE approaches are more appropriate than is the APV approach when a firm focuses on a target debt-value ratio.